Quantum Chromo Dynamics

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Overview

I. Quarks and the Standard Model

II. The Birth of Quantum Chromodynamics

III. Basic Properties of QCD

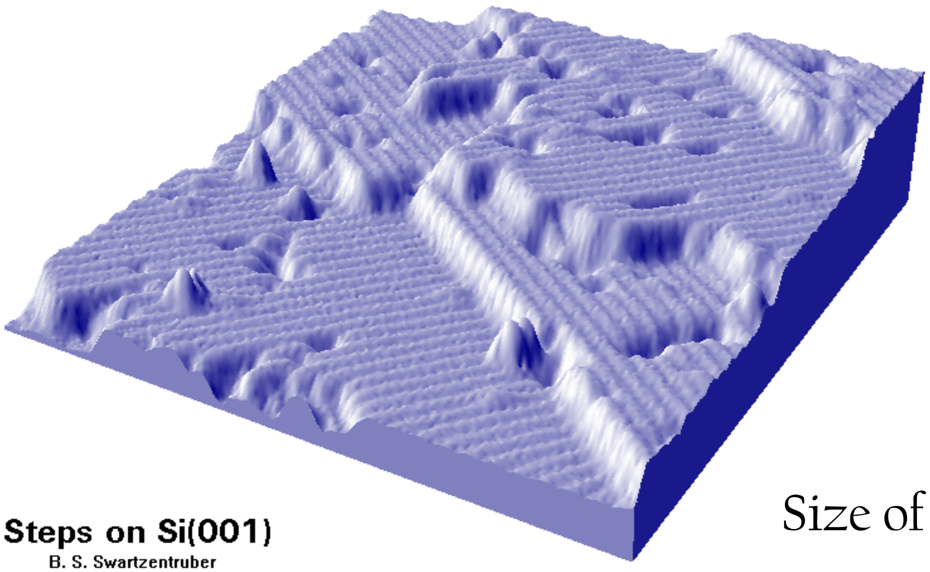
IV. The Cosmic Connection

V. High Energy Heavy Ion Collisions

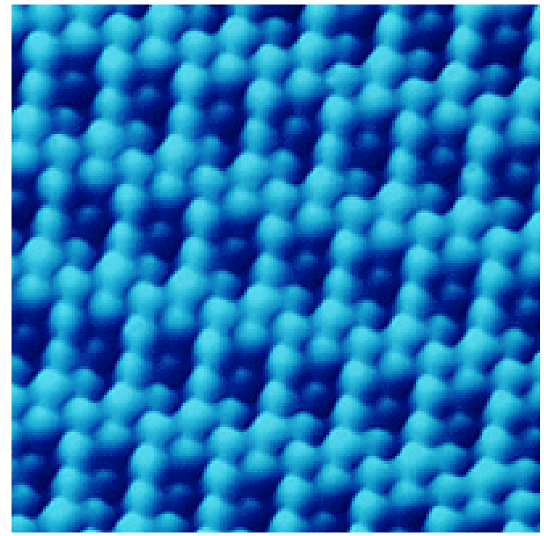
I. Quarks and the Standard Model

Atoms

Atoms: the building blocks of matter.
Today: we can make atoms visible



Steps on Si(001)
B. S. Swartzentruber
Sandia National Lab



U of Oregon Chemistry

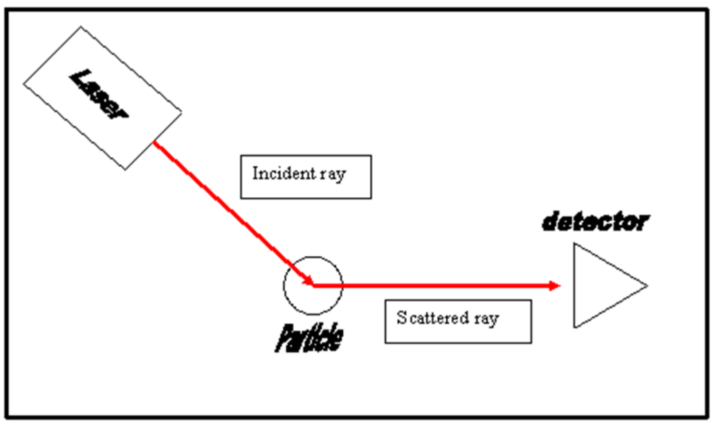
Size of the smallest atom (hydrogen):

0.000 000 000 1 m (meter)
= 10^{-10} m = 1 Angstrom

How is it possible to see such tiny structures?

Scattering: Gateway to the Subatomic World

Our vision: the eye collects light reflected from objects and our brain processes the information

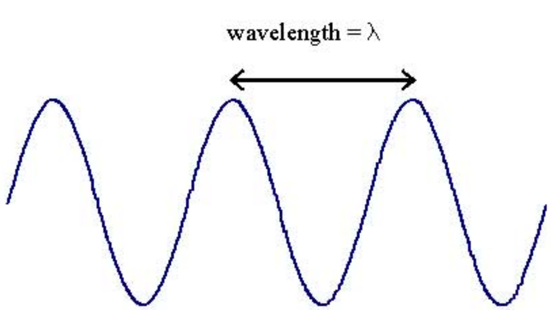


Light: wavelength 4000 – 7000 Angstrom, too large to see an atom.
Better: X-rays, electrons

Use this principle:
Shoot a ray of light or particles at an object.
Measure the scattered rays with a detector.

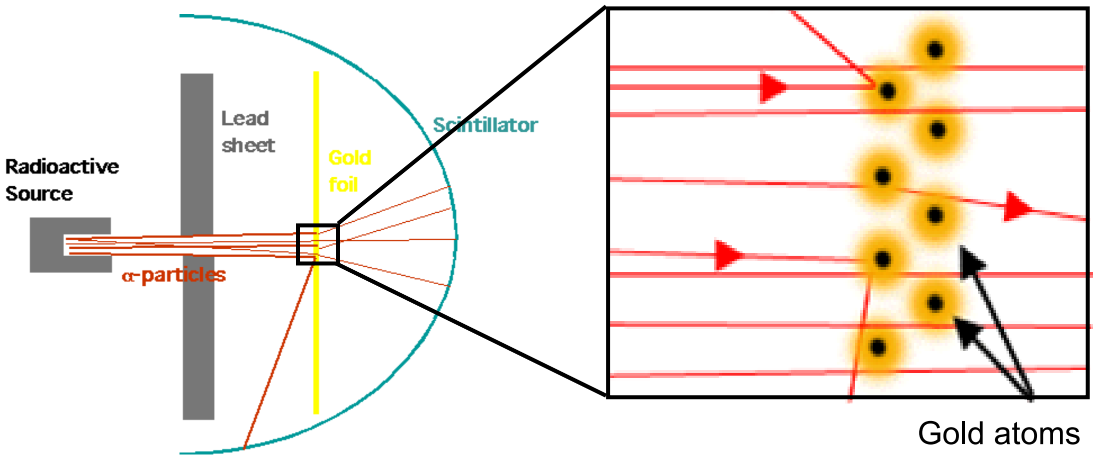
Resolution of the probe (light, particle) is important:

The wavelength must be smaller than the size of the structure to probe.

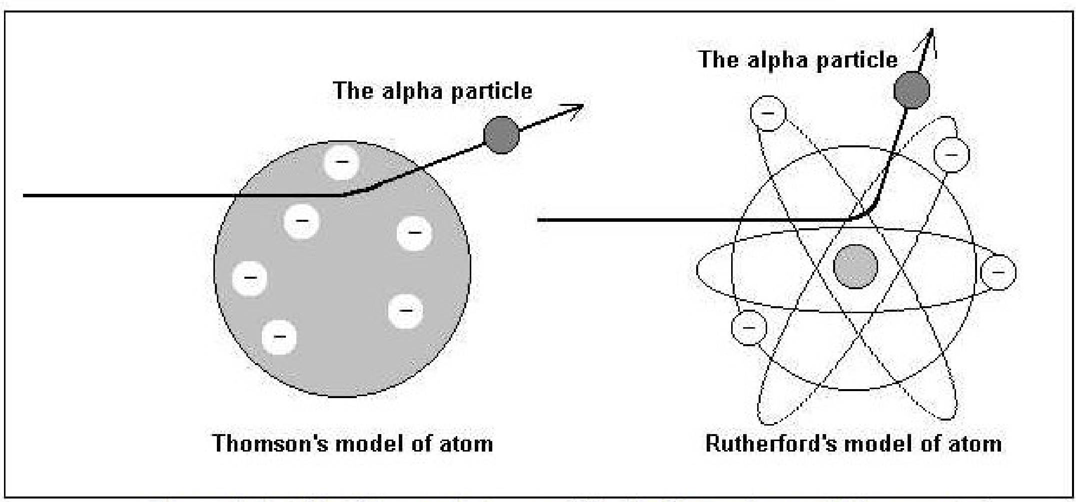


The Rutherford Experiment

In 1911 E. Rutherford carried out his famous experiment with α-particles . His target were gold atoms.



Rutherford's result indicated that atoms are mostly empty space with a small massive center!



The models of the Thomson's atom and Rurtherford's atom; and the expected aberrations of alpha particle in both cases.

The positive charge in an atom and most of its mass is concentrated in a tiny, very dense center, the nucleus.

Particles

We distinguish particles by their …

participation in strong interactions
YES: they are called *hadrons* (or *quarks*)
e.g. proton, neutron
NO: they are called *leptons*
e.g. electron

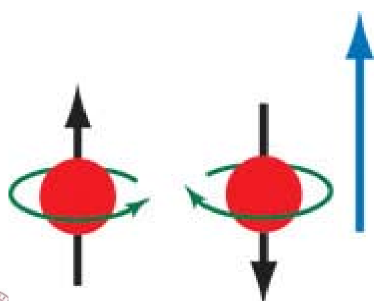
electric charge
positive or negative
usually in multiples of e

mass
usually measured in electronvolts (eV)
1 u ~ 0.939 GeV (Gigaelectronvolts,
Giga = Billion)

spin
= Quantized angular momentum
(can take values 0$\hbar$, ½ $\hbar$, 1 $\hbar$, 3/2 $\hbar$, 2 $\hbar$, etc)
Electrons, protons, neutrons: spin ½ $\hbar$

Particles with integer spin are called *bosons*.

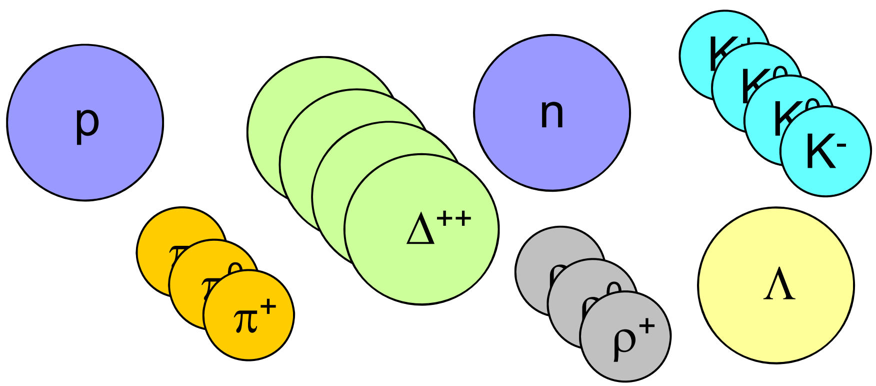
Particles with half-integer spin are called *fermions*.



Electrons, protons and neutrons are fermions.

The Hadron Zoo

In 1940 only 5 elementary particles were known: protons, neutrons, electrons, muons and positrons. Only protons and neutrons are hadrons (the strong force acts on them).



With the advent of accelerators at the end of the decade a big 'zoo' of hadrons was discovered:
Pions, kaons, rhos, ... many more

They could be grouped into one of two categories:

- Heavier ***baryons***, whose total number is always conserved.
  E.g. protons, neutrons
- Lighter ***mesons***, which can decay into particles which are not hadrons.
  E.g. pions, kaons

Too many! Maybe hadrons are not elementary particles after all?

The Standard Model

- In the 1960s and 1970s the Standard Model of Particle Physics was developed.
- Hadrons are bound states of new fermions called “quarks”.
- Besides the well-understood electromagnetic force there is a weak (nuclear) force and a strong (nuclear) force.
- All 3 forces are described by gauge fields with gauge symmetry groups U(1), SU(2) and SU(3) .
- Quantum Field Theory of the strong force = **Q**uantum **C**hromo**D**ynamics (QCD)
- The Higgs mechanism was introduced to break the electroweak symmetry and give masses to the weak force carriers.

The Standard Model

BOSONS force carriers
spin = 0, 1, 2, …

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W$^-$	80.39	–1
W$^+$ W bosons	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin =1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

6 fermions and 6 leptons come in 3 identical generations (only masses are different) Plus they have antiparticles.

New: Higgs boson.

FERMIONS matter constituents
spin = 1/2, 3/2, 5/2, …

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	(0−0.13)×10^{-9}	0
e electron	0.000511	–1
ν_M middle neutrino*	(0.009−0.13)×10^{-9}	0
μ muon	0.106	–1
ν_H heaviest neutrino*	(0.04−0.14)×10^{-9}	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

Leptons and quarks feel the weak force. Only quarks have color charges and feel the strong force.

II. The Birth of QCD

Start with Electrodynamics

- Maxwell: $\partial_\mu F^{\mu\nu} = 0$

- Field strength:

$$F^{\mu\nu} = \frac{i}{e}\left[D^\mu, D^\nu\right] = \partial^\mu A^\nu - \partial^\nu A^\mu$$

Commutator!

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix}$$

- Vector potential: $A^\mu = \left(\phi, \vec{A}\right)$

- Covariant derivative:

$$D^\mu = \partial^\mu - ieA^\mu$$

- U(1) gauge invariance:

$$A^\mu(x) \to A^\mu(x) - \frac{1}{e}\partial^\mu \Lambda(x)$$

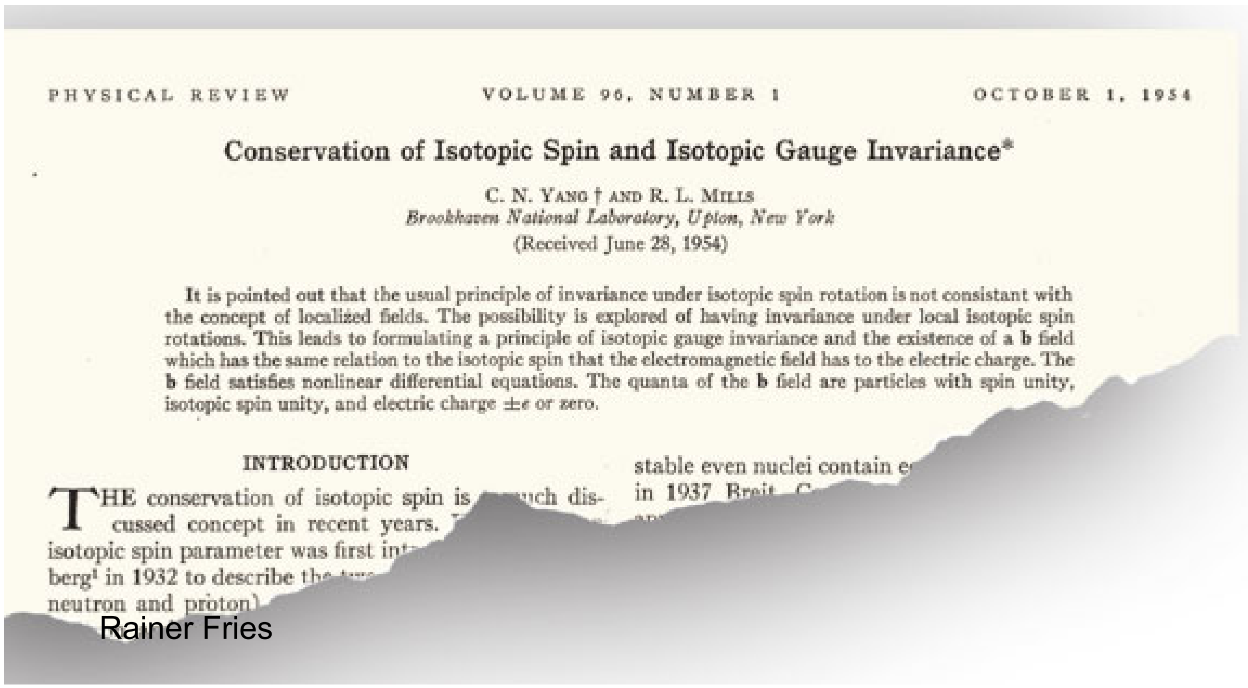
Gauge group determined by

$$e^{i\Lambda(x)} \in U(1)$$

- Lagrangian: $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

A Strange New Electrodynamics

- C N Yang & R L Mills (1954) worked out the math for a generalization to "gauge fields" with more cimplicates symmetry groups.
  - Most important example: $SU(N)$ = unitary N xN matrices with determinant 1.
- From now on $e^{i\Lambda(x)} \in SU(N)$ where $\Lambda(x)$ is a function that takes values in the space of N xN matrices.


PHYSICAL REVIEW VOLUME 96, NUMBER 1 OCTOBER 1, 1954

Conservation of Isotopic Spin and Isotopic Gauge Invariance*

C. N. Yang † and R. L. Mills
Brookhaven National Laboratory, Upton, New York
(Received June 28, 1954)

It is pointed out that the usual principle of invariance under isotopic spin rotation is not consistant with the concept of localized fields. The possibility is explored of having invariance under local isotopic spin rotations. This leads to formulating a principle of isotopic gauge invariance and the existence of a **b** field which has the same relation to the isotopic spin that the electromagnetic field has to the electric charge. The **b** field satisfies nonlinear differential equations. The quanta of the **b** field are particles with spin unity, isotopic spin unity, and electric charge ±e or zero.

INTRODUCTION

THE conservation of isotopic spin is much dis- cussed concept in recent years. isotopic spin parameter was first in berg[1] in 1932 to describe the neutron and proton)

stable even nuclei contain in 1937 Breit

SU(N) Yang-Mills Fields

- Maxwell: $\partial_\mu F^{\mu\nu} = 0$
- Field strength:

$$F^{\mu\nu} = \frac{i}{e}\left[D^\mu, D^\nu\right] = \partial^\mu A^\nu - \partial^\nu A^\mu$$

- Vector potential: $A^\mu = \left(\phi, \vec{A}\right)$
- Covariant derivative:

$$D^\mu = \partial^\mu - ieA^\mu$$

- U(1) gauge invariance:

$$A^\mu(x) \to A^\mu(x) - \frac{1}{e}\partial^\mu \Lambda(x)$$

- Lagrangian: $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

- Yang-Mills: $D_\mu F^{\mu\nu} = 0$
- Field strength:

$$F^{\mu\nu} = \frac{i}{e}\left[D^\mu, D^\nu\right] = \partial^\mu A^\nu - \partial^\nu A^\mu - ig\left[A^\mu, A^\nu\right]$$

- Vector potential: $A^\mu = \left(\phi, \vec{A}\right)$
- Covariant derivative:

$$D^\mu = \partial^\mu - igA^\mu$$

- $SU(N)$ gauge invariance:

$$A^\mu \to e^{i\Lambda} A^\mu e^{-i\Lambda} - \frac{i}{g}\left(\partial^\mu e^{i\Lambda}\right)e^{-i\Lambda}$$

- Lagrangian: $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

SU(N) Yang-Mills Fields

- All fields A and F and the current J are now N x N matrices.
- g = coupling constant of the theory.
- Non-abelian symmetry group ⇒ "non-abelian gauge field"
- One immediate consequence: quadratic and cubic terms in the equations of motion! The field theory is non-linear.

$$\sim \partial_\mu \left[A^\mu, A^\nu\right], \quad \sim A_\mu \left[A^\mu, A^\nu\right]$$

- Yang-Mills: $D_\mu F^{\mu\nu} = 0$
- Field strength:

$$F^{\mu\nu} = \frac{i}{e}\left[D^\mu, D^\nu\right] = \partial^\mu A^\nu - \partial^\nu A^\mu - ig\left[A^\mu, A^\nu\right]$$

- Vector potential: $A^\mu = \left(\phi, \vec{A}\right)$
- Covariant derivative:

$$D^\mu = \partial^\mu - igA^\mu$$

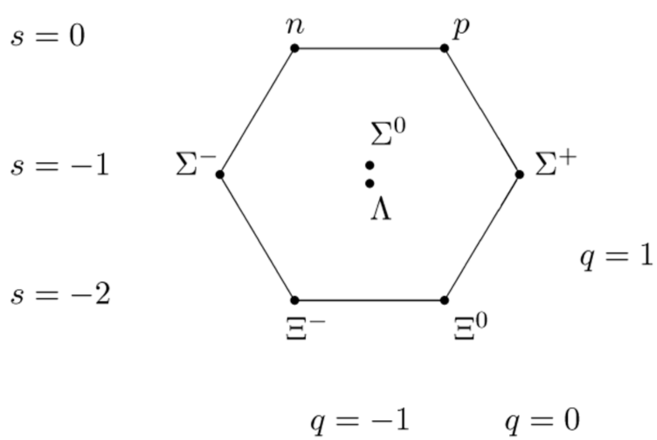
- $SU(N)$ gauge invariance:

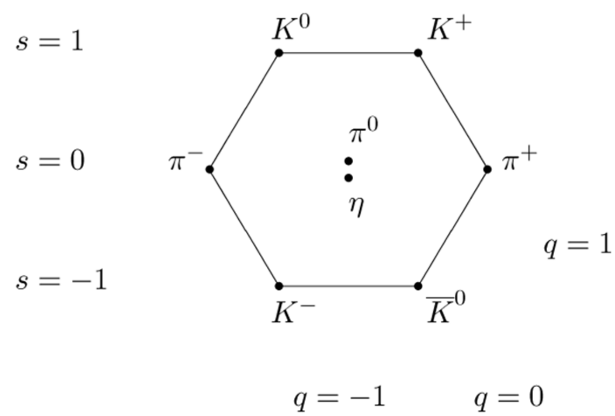
$$A^\mu \rightarrow e^{i\Lambda} A^\mu e^{-i\Lambda} - \frac{i}{g}\left(\partial^\mu e^{i\Lambda}\right) e^{-i\Lambda}$$

- Lagrangian: $-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

Taming the Hadron Zoo

- Hundreds of hadrons. Who ordered that?
- Gell-Mann & Zweig (1964): the zoo of hadrons could be understood if hadrons consisted of combinations of more fundamental spin-1/2 fermions with SU(N_f) flavor symmetry. Gell-Mann called them quarks.





- A crazy idea at the time!

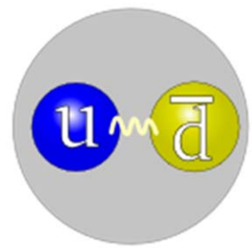
Gell-Mann: 'Such particles [quarks] presumably are not real but we may use them in our field theory anyway.'

Hadrons as Bound States

Postulate a new quantum number: color (“chromos”). Quarks carry one of 3 colors.

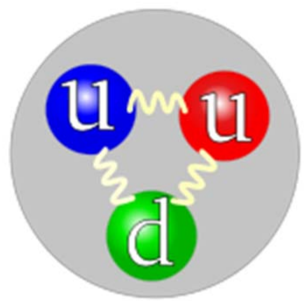
Hadrons are color neutral, i.e. the color of the quarks and gluon inside has to add up to ‘white’.

- Meson = quark + antiquark



□+

- Baryon = 3 quarks



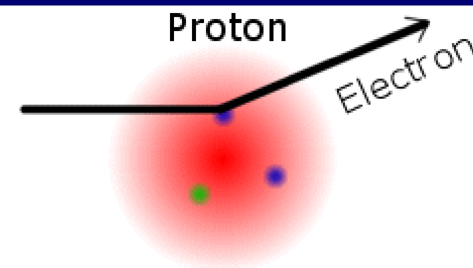
p

Those quarks are called the valence quarks of a hadron.

E.g. the valence quark structure of the proton is *uud*

Dissecting Hadrons

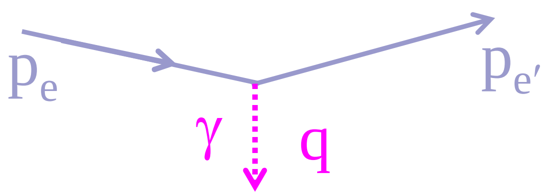
- An new Rutherford experiment at higher energy:



- Cross section for inelastic e+p scattering: extract two "structure functions" F_1 and F_2.

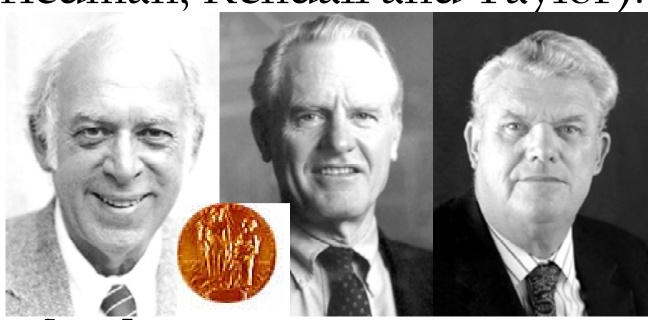
$$\frac{d\sigma}{dE'd\Omega} = \left(\frac{\alpha\hbar}{2E\sin^2(\theta/2)}\right)^2 \left[\frac{2F_1(x,Q^2)}{M}\sin^2(\theta/2) + \frac{2MxF_2(x,Q^2)}{Q^2}\cos^2(\theta/2)\right]$$

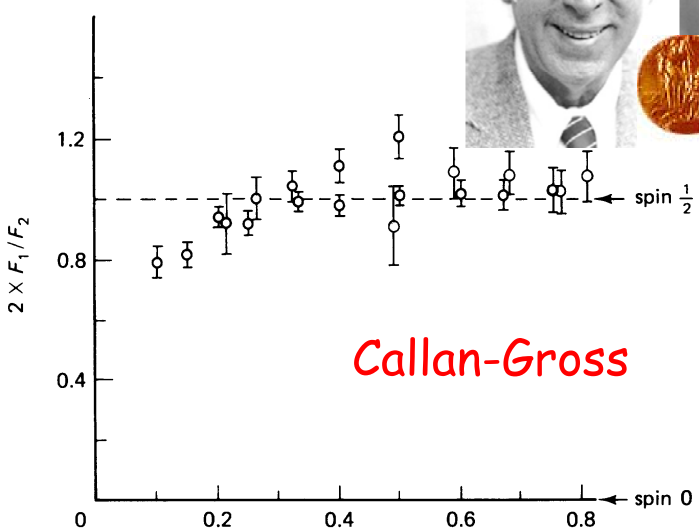
  - Simply given by leading order (one-photon exchange) QED and Lorentz invariance.
- Two independent kinematic variables:
  - $Q^2 = -(p'_e - p_e)^2$ is the virtuality of the exchanged photon
  - x is the momentum fraction of the object inside the proton struck by the photon (elastic scattering: $x = 1$)
  - They can be related to the observables: the deflection angle and the energy loss of the electron.



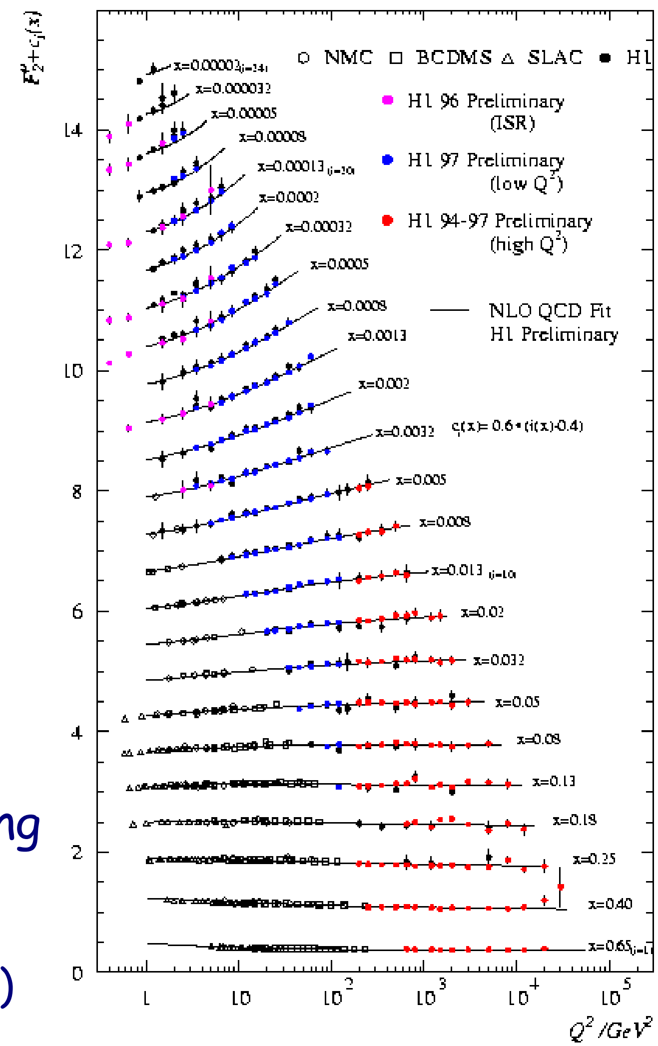
Quarks

- Different predictions had been made.
- Suppose the proton consists of point-like spin-½ fermions (as in the quark model). Then:
  - F_1, F_2 don't depend on Q^2 (Bjorken scaling)
  - F_1, F_2 are not independent: $2xF_1 = F_2$ (Callan-Gross relation)
- SLAC, 1968 (Friedman, Kendall and Taylor): Quarks it is!





Callan-Gross

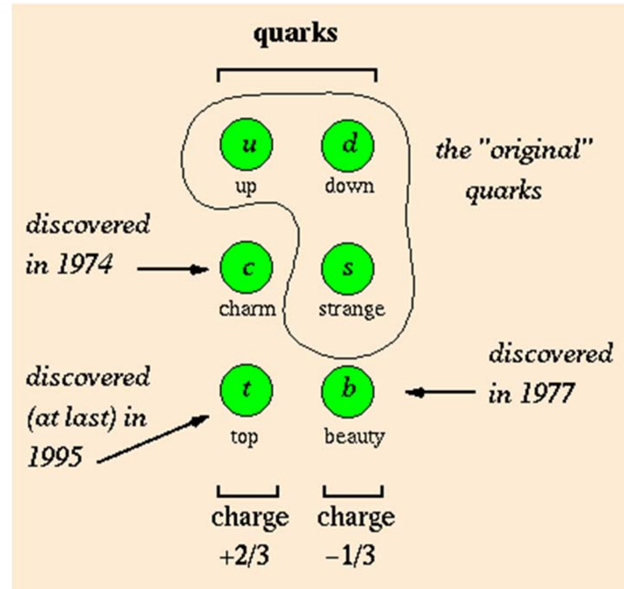


Bjorken scaling

(shown here for HERA data)

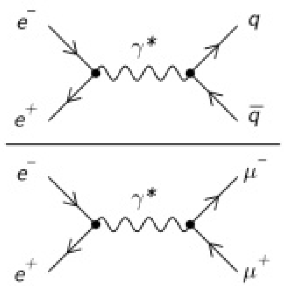
Quarks

- The complete quark family:
- What are their interactions?



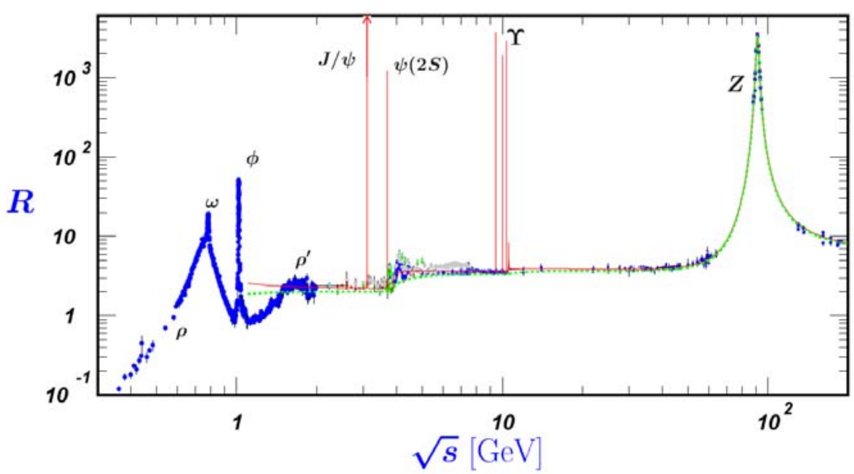
- e+e- collisions: each quark comes in triplicate!

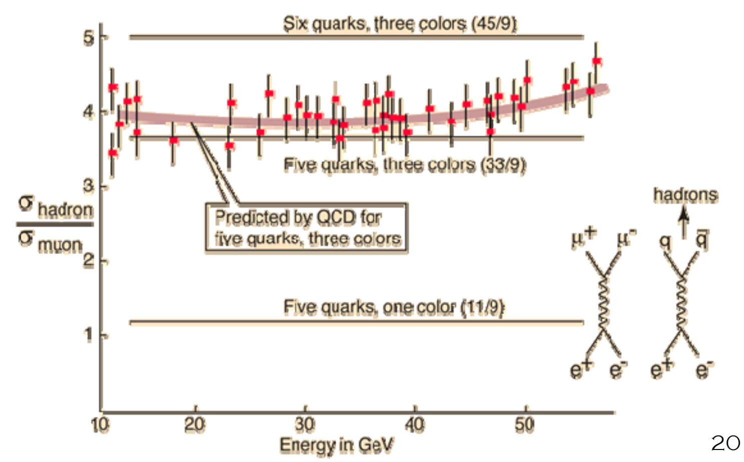
$$R = \frac{\sigma(e^-e^+ \to \text{hadrons})}{\sigma(e^-e^+ \to \mu^-\mu^+)} \approx$$



$$= \sum_f \left(\frac{q_f}{e}\right)^2$$

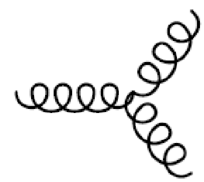
- Confirm there is a new quantum number: color!





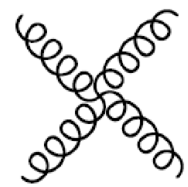
Quantum Chromodynamics

- Fritsch, Gell-Mann, Leutwyler (1972): Quarks couple to a SU(N_c) Yang-Mills field. N_c = number of colors.
  - Quanta of the Yang-Mills/gauge field: gluons
  - Color plays the role of the “charge” of the quark field.
- Quantum chromodynamics is born!
- QCD Lagrangian: $$L = \sum_f \bar{q}_f \left(i\gamma_\mu D^\mu - m_f \right) q_f - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$
  - q = N_f quark fields of masses m_f. F = gluon field strength.
- Quantization: non-linearity $\rightarrow$ self-interaction of the gluon field
  - Gluon itself carries N_c^2-1 colors.
  - 3-gluon vertex $-\frac{g}{2} f_{abc} \left(\partial^\mu A_a^\nu - \partial^\nu A_a^\mu \right) A_\mu^b A_\nu^c$



$\sim g$

  - 4-gluon vertex $-\frac{g^2}{4} f_{abc} f_{cde} A_{a\mu} A_{b\nu} A_c^\mu A_d^\nu$



$\sim g^2$

III. Basic Properties of QCD

Can We Solve QCD?

QCDOC at Brookhaven National Lab

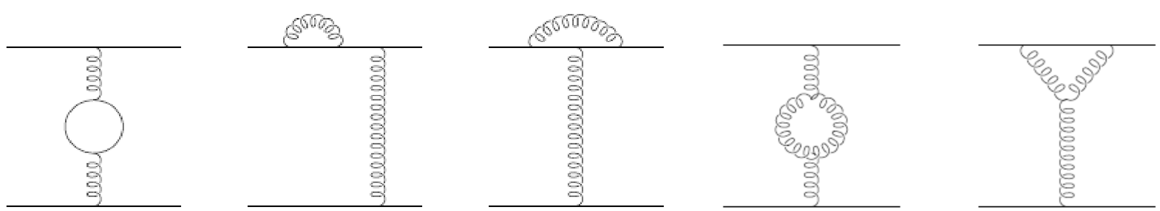
- Analytically: No!
- Numerically: Yes, in certain situations → Lattice QCD.
  - Discretize space-time and use euclidean time.
  - Extremely costly in terms of CPU time, very smart algorithms needed.
- Perturbation theory: only works at large energy scales / short distances (see asymptotic freedom below).
- Effective theories: Based on certain approximations of QCD or general principles and symmetries of QCD (e.g. chiral perturbation theory, Nambu-Jona Lasinio (NJL) model, classical QCD etc.)

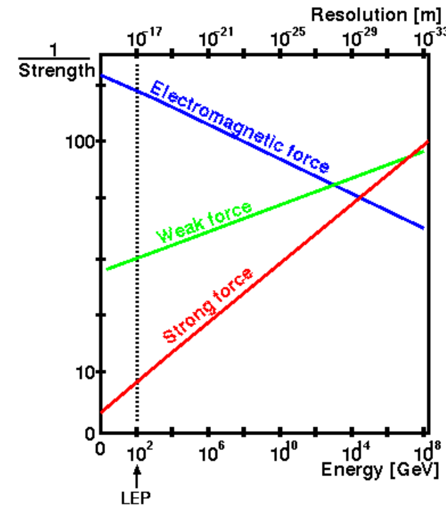
Asymptotic Freedom

- Running coupling in perturbative QCD (pQCD): $\mu \frac{dg}{d\mu} = -\beta_1 g^3 - \beta_2 g^3 - \ldots$
  - Perturbative β-function known up to 4 loops.

β-function

- Leading term in pQCD $\beta_1 = \frac{1}{(4\pi)^2}\left(11 - \frac{2}{3}N_f\right) \rightarrow \beta < 0$
  - For any reasonable number of active flavors $N_f = 3 \ldots 6$.
  - E.g. from pQCD "potential" (cf. Handbook of Perturbative QCD)





- In QED: $\mu \frac{de}{d\mu} = \frac{1}{12\pi^2} e^3 + \ldots > 0$

QED: *e* larger at higher energies/smaller distances:
screening through electron-positron cloud

QCD: *g* smaller at higher energies/smaller distances:
anti-screening through gluon loops

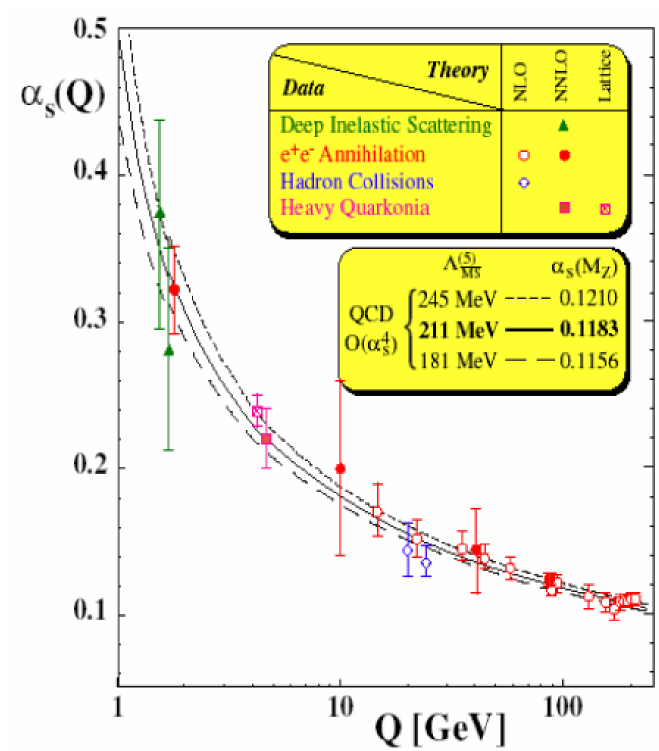
Asymptotic Freedom

- Leading order running of the coupling:
  - Λ_{QCD} here: integration constant; “typical scale of QCD”
  - $\Lambda_{QCD} \cong 200$ MeV

$$\alpha_s = \frac{4\pi}{\beta_1 \ln \frac{\mu^2}{\Lambda_{QCD}^2}}$$

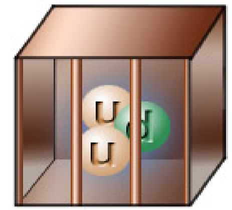
- Vanishing coupling at large energies = Asymptotic Freedom
  - This permits, e.g., the application of pQCD in DIS.
  - Large coupling at small energy scales = “infrared slavery”
  - Bound states can not be treated perturbatively

Gross, Wilczek, Politzer (1974)

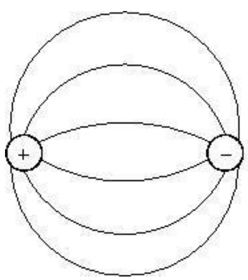


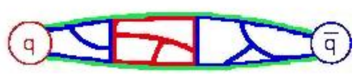
Confinement

- Experimental fact: no free quarks or fractional charges found.



- Confinement property of QCD:
  - Only color singlet configurations allowed to propagate over large distances.
  - Energy required to remove a quark larger than 2-particle creation threshold.
- Heuristic picture:
  - At large distances the Coulomb-like gluon field between quarks becomes a flux tube with string-like properties.



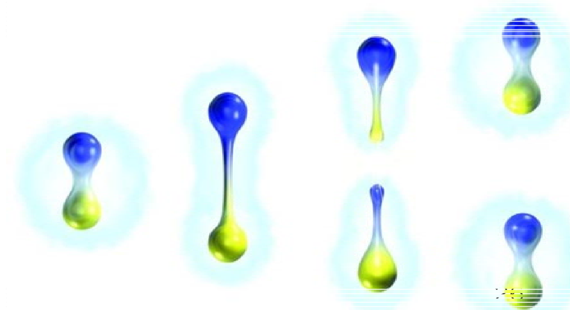


Confinement is non-perturbative. It has not yet been fully understood.

It has been named one of the outstanding mathematical problems of our time. The Clay Foundation will pay you $1,000,000 if you solve it!

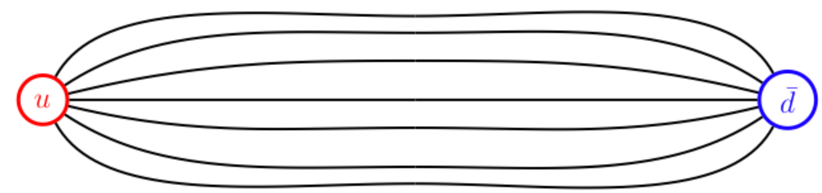
http://www.claymath.org/

  - String breaks once enough work is done for pair creation.
  - Flux tubes can be understood as gluon flux expelled from the QCD vacuum.



Confinement

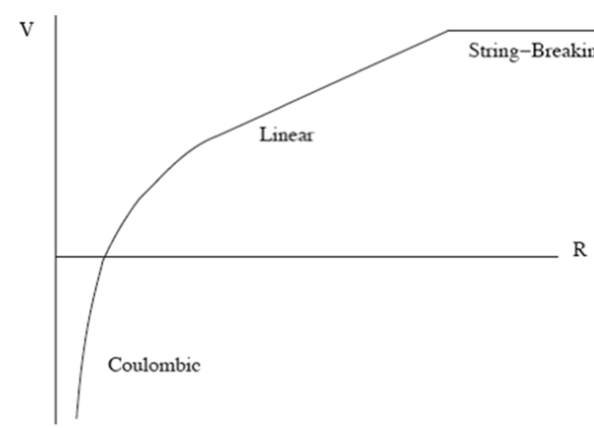
- Why gluon flux tubes?
  - Anti-screening of color charges from perturbative running coupling: Dielectric constant of QCD vacuum $\varepsilon < 0$.
  - Dual Meissner Effect: $\varepsilon \to 0$ for long distances, expelling (color) electric flux lines.
  - Usual Meissner Effect in superconductors: perfect diamagnetism expels magnetic flux.



- Potential between (heavy) quarks can modeled successfully with a Coulomb plus linear term:

$$V(r) = -\frac{a}{r} + Kr$$

  - String tension $K \cong 0.9$ GeV/fm.
  - Successful in quarkonium spectroscopy.
  - Can be calculated in lattice QCD (later).



Global Symmetries

- Classical QCD has several gobal symmetries.
- Chiral symmetry $SU(N_f)_L \times SU(N_f)_R$:
  - $e^{i\Lambda_L} \otimes e^{i\Lambda_R}$ acting on $2N_f$-tuple $(q_{L,f}, q_{R,f})$ of left/right-handed quarks $q_{R,L} = \frac{1}{2}(1 \pm \gamma^5)q$
  - Obvious when QCD Lagrangian rewritten with right/left-handed quarks :

$$L = \sum_f \bar{q}_{L,f} i\gamma_\mu D^\mu q_{L,f} + \sum_f \bar{q}_{R,f} i\gamma_\mu D^\mu q_{R,f} - \sum_f m_f \bar{q}_{L,f} q_{R,f} - \sum_f m_f \bar{q}_{R,f} q_{L,f} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

  - Chiral symmetry slightly broken explicitly by finite quark masses of a few MeV.

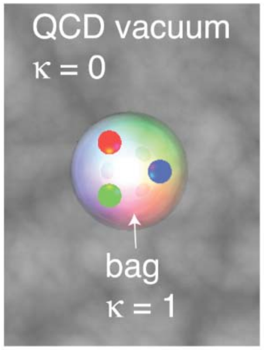
- Scale invariance: massless classical QCD does not have a dimensionful parameter.
- Both symmetries are broken:
  - Chiral symmetry is spontanteously broken in the ground state of QCD by a chiral condensate $\langle q\bar{q} \rangle$.
  - Quantum effects break scale invariance: Λ_{QCD} is scale intrinsic to QCD.

QCD Vacuum

- Pions are the Goldstone bosons from the spontaneous breaking of chiral symmetry.
- Gell-Man-Oaks-Renner relation: $f_\pi^2 m_{\pi^\pm}^2 = -\frac{m_u + m_d}{2} \langle u\bar{u} + d\bar{d} \rangle$
- Infer value of chiral condensate at T=0: $\langle q\bar{q} \rangle = \frac{1}{2} \langle u\bar{u} + d\bar{d} \rangle \approx -(250\ \text{MeV})^3$
  - Chiral perturbation theory: $\langle q\bar{q} \rangle$ decreasing with increasing temperature.
- There is also a gluon condensate in the QCD vacuum $\left\langle \frac{\alpha_s}{\pi} F_{\mu\nu} F^{\mu\nu} \right\rangle \approx (300\ \text{MeV})^4$
- Dilation current θ^μ from scale invariance:
  - Conserved for scale-invariant QCD (Noether Theorem).
  - Through quantum effects: $\partial_\mu \theta^\mu = T_\mu^\mu = \frac{\beta}{2g} F_{\mu\nu} F^{\mu\nu}$
  - Gluon condensate implies a non-vanishing energy momentum tensor of the QCD vacuum!

QCD Vacuum

- Assuming for vacuum $T^{\mu\nu} = \varepsilon_{\text{Vac}}\, g^{\mu\nu}$ from Lorentz invariance.
- Energy density of the vacuum: $\varepsilon_{\text{Vac}} = -300\,\frac{\text{MeV}}{\text{fm}^3} \equiv -B$
- This is also called the Bag Constant for a successful model for hadrons: vacuum exerts a positive pressure *P=B* onto a cavity with quark modes.



- Summary: QCD vacuum is an ideal (color) dielectric medium with quark and gluon condensates, enforcing confinement for all but color singlet configurations.

Why Quark Gluon Plasma?

- Collins and Perry, 1975: Due to asymptotic Freedom coupling becomes arbitrarily weak for large energies i.e. also for large temperatures.
- Therefore quarks and gluons should be asymptotically free at very large temperatures T.
- This hypothetical state without confinement at high T would be called Quark Gluon Plasma (QGP).
- Expect vacuum condensates to melt as well $\rightarrow$ chiral symmetry restoration at large T.
- How can confinement be broken?

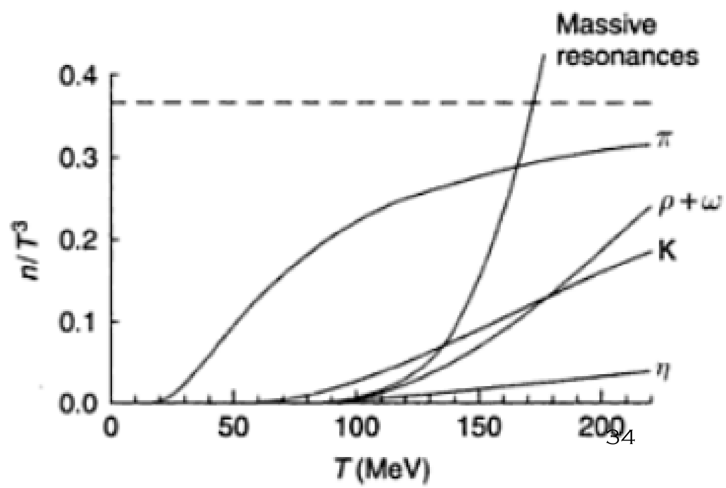
1.3 Free the Quarks!

END OF CONFINEMENT: PERCOLATION

- Hadronic states cease to exist due to percolation.
- Density of massless free pion gas $n_\pi(T) = d_\pi \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{k/T}-1} = \frac{3\zeta(3)}{\pi^2} T^3$ from Bose distribution growing like T^3.
- Pions will start to overlap at some temperature!
- Free pion volume $V_\pi = \frac{4\pi}{3} R_\pi^{\ 3}$ with $R_\pi \cong 0.65$ fm
- Closest packing of pions corresponds to ~ 260 MeV, percolation at $n_\pi V_\pi = 0.35$ which corresponds to T_c = 186 MeV.
- Consequences:
  - Individual hadrons no longer well defined.
  - Percolation would allow quarks to propagate over large distances avoiding the QCD vacuum, confinement broken.
- A very simple model (massless free pions!),which gives a fair estimate of T_c.

End of Confinement: Hagedorn

- Hadronic physics breaks down at some temperature due to the "Hagedorn Catastrophe".
- Above $T \cong 100$ MeV thermal excitations of hadrons besides pions are important.
  - Many resonances contribute, each suppressed by factor $e^{-M/T}$ at large mass M.
- Hadron spectrum at large mass:
  - Let $\rho(M)dM$ be the number of resonances in a mass interval *dM*.
  - Fit to hadron spectrum: exponential increase in states parameterized as $\rho(M) = \frac{A}{M^{\alpha}} e^{M/T_0}$
- Total density of hadrons $n_{\text{tot}}(T) = \int dM \rho(M) n(M,T)$ diverges for $T > T_0$.
- Hadron thermodynamics stops above $T_0 \cong 150 \ldots 200$ MeV.



From YHM;
originally: Gerber and Leutwyler;
pions: chiral perturbation theory

QGP as a Relativistic Free Gas

- Start with relativistic free gas of massless pions; degeneracy $d_\pi = 3$ (isospin)

$$P_\pi = d_\pi \frac{\pi^2}{90} T^4 \qquad \varepsilon_\pi = d_\pi \frac{\pi^2}{30} T^4 \qquad s_\pi = d_\pi \frac{4\pi^2}{90} T^4$$

  - E.g. P and ε from distributions fcts. f via energy momentum tensor: $T^{\mu\nu} = \int \frac{d^3k}{(2\pi)^3} \frac{k^\mu k^\nu}{k^0} f(k)$
  - Entropy s from thermodynamic relation $Ts = \varepsilon + P$.
  - Note $\varepsilon = 3P$, speed of sound $c_s^2 = \partial P / \partial \varepsilon = 1/3$.
  - Corrections through interactions: cf. YHM, ch. 3.6

- QGP: $P_{\mathrm{QGP}} = d_{\mathrm{QGP}} \frac{\pi^2}{90} T^4 - B \qquad \varepsilon_{\mathrm{QGP}} = d_{\mathrm{QGP}} \frac{\pi^2}{30} T^4 + B \qquad s_{\mathrm{QGP}} = d_{\mathrm{QGP}} \frac{4\pi^2}{90} T^4$

  - Bag constant B: measure relative to vacuum.
  - Degeneracy: $d_{\mathrm{QGP}} = 2 \times (N_c^2 - 1) + 2 \times 2 \times \frac{7}{8} \times N_c \times N_f = 16 + \frac{21}{2} N_f$
  - $d_{\mathrm{QGP}} = 37$ for two light flavors, $d_{\mathrm{QGP}} = 47.5$ for three light flavors.

- Massive increase in degrees of freedom from hadron gas to QGP.

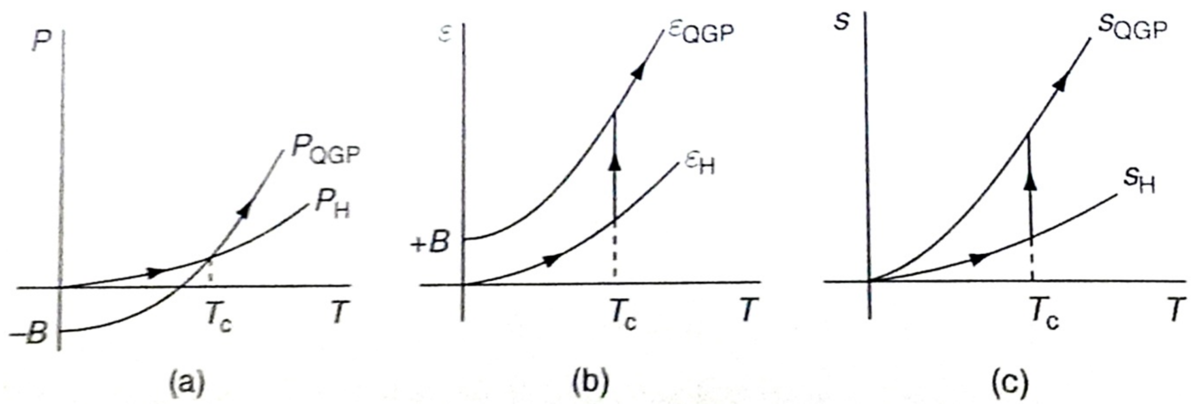
A Simple Equation of State

- Assume free pion gas and free quark gluon gas as two phases.
  - Low T: $P_\pi > P_{QGP}$ due to bag constant → pion gas preferred state
  - High T: $P_\pi < P_{QGP}$ due to larger degeneracy in QGP → QGP preferred state
- Phase transition: phase equilibrium requires $P_\pi = P_{QGP}$ $\Rightarrow T_c = \sqrt[4]{\frac{90}{\pi^2}\frac{B}{d_{QGP}-d_\pi}}$
- With $B = (220\text{ MeV})^4$ and $N_f = 2$: $T_c \cong 160$ MeV.

- While P is continuous, ε and s exhibit a jump at T_c → first order phase transition.
- Latent heat

$$H = 4B^{1/4}$$

From YHM



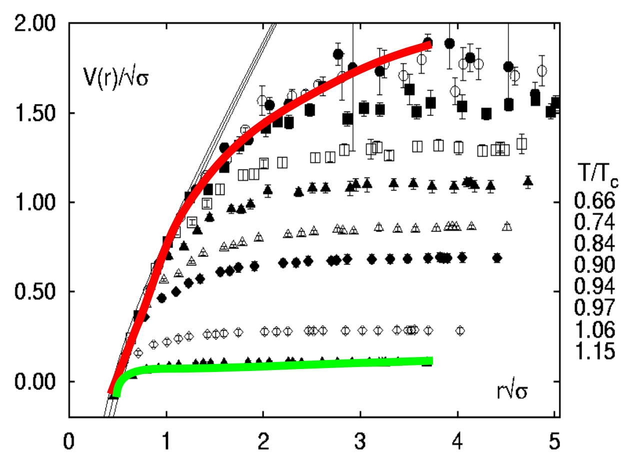
- So far zero baryon chemical potential $\mu = 0$ (i.e. equal numbers of quark and antiquarks in equilibrium).

QGP: Survey of Lattice QCD Results

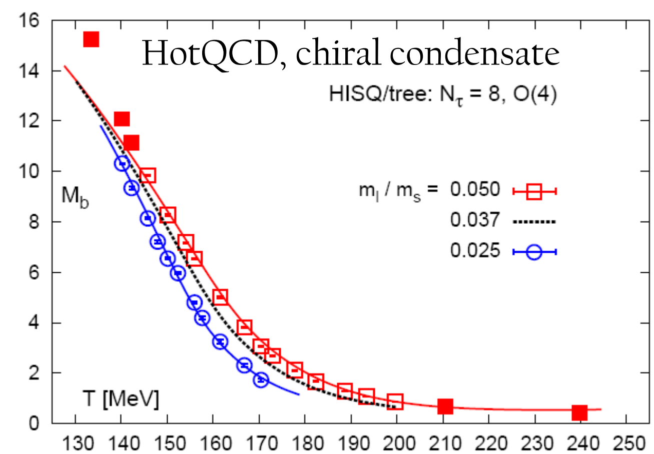
- Currently best method to determine the QCD equation of state: lattice QCD
  - Experiment: high energy heavy ion collisions

- Chiral condensate = order parameter for chiral phase transition.
- Transition temperature for chiral phase transition:
  - T_c = 154±9 MeV [RBC-Bielefeld]
  - T_c = 151 MeV [Wuppertal-Bielefeld]

- Static heavy quark potential: QCD strings melt

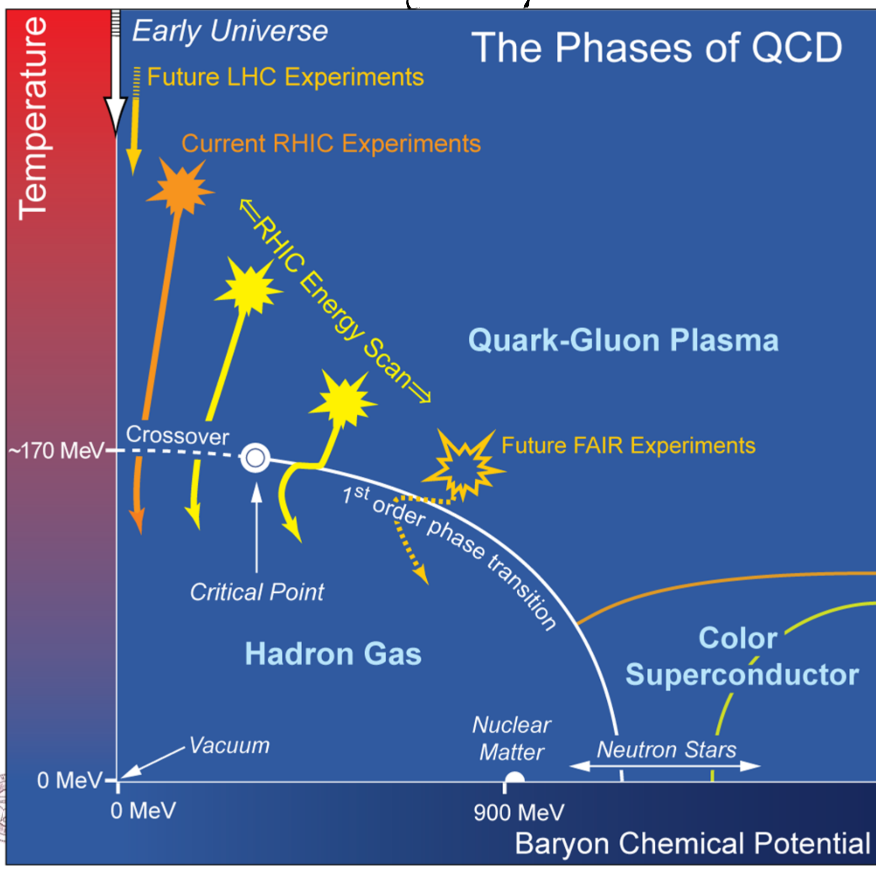


Static quark potential (Karsch et al.)

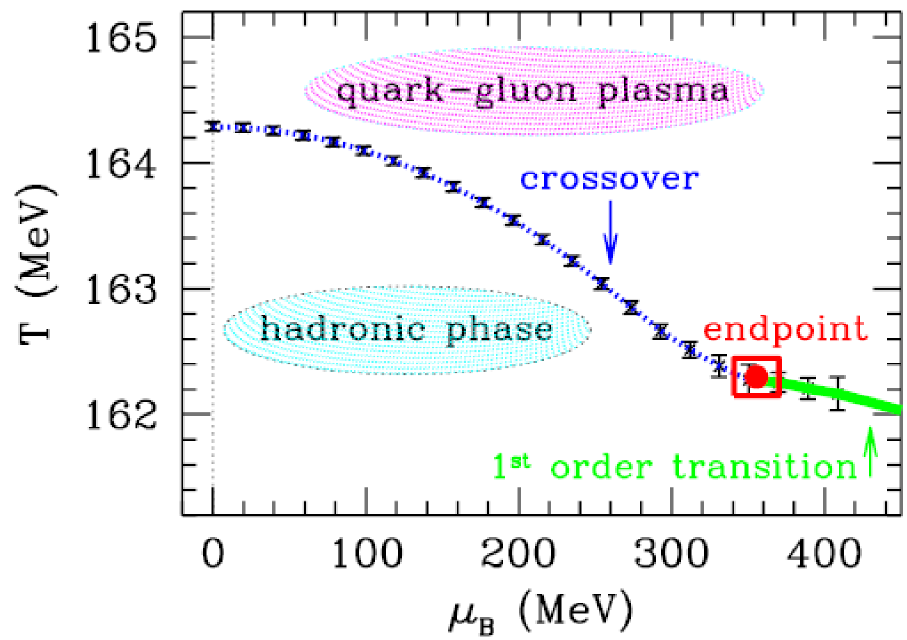


QGP: Survey of Lattice QCD Results

- Phase diagram of QCD in T-μ plane: away from μ=0 very little known.
- Sign problem for $\mu \neq 0$ in lattice QCD.
  - Need innovative techniques: reweighting, Taylor expansion, imaginary μ, ...
- Probably a critical point (endpoint of 1st order phase transition line) located close to $T = T_c$ and $\mu \sim$ 200-400 MeV.



[Fodor, Katz, JHEP 04, 050 (2004)]

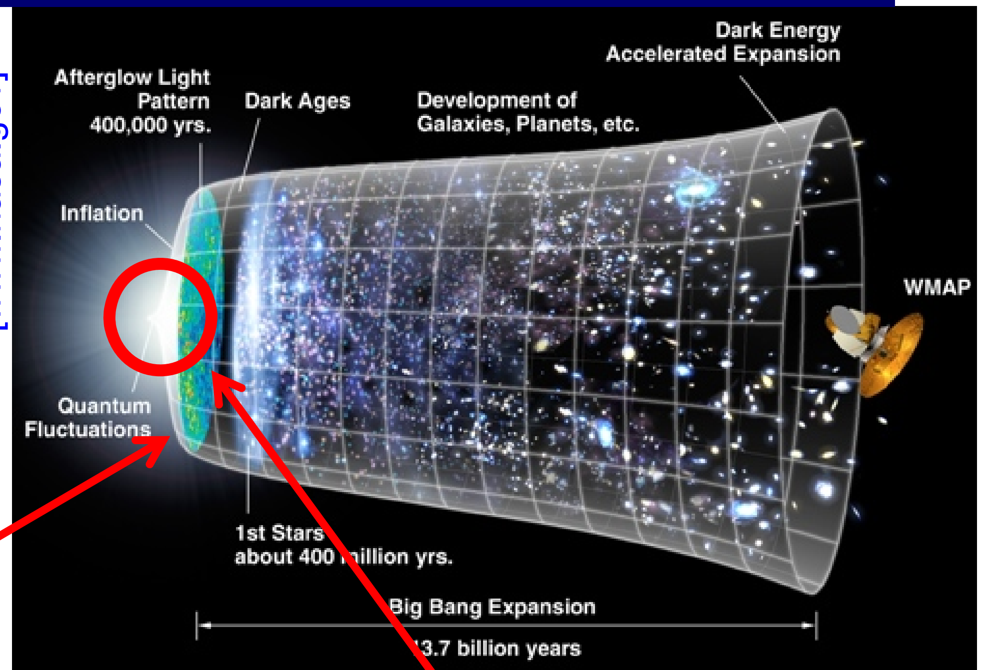


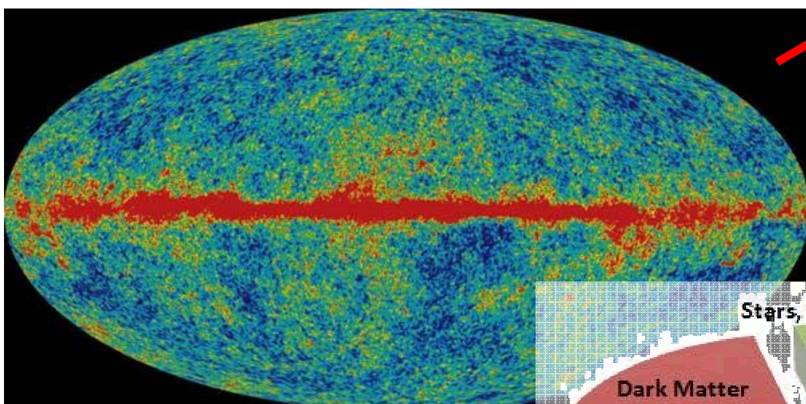
IV. The Cosmic Connection

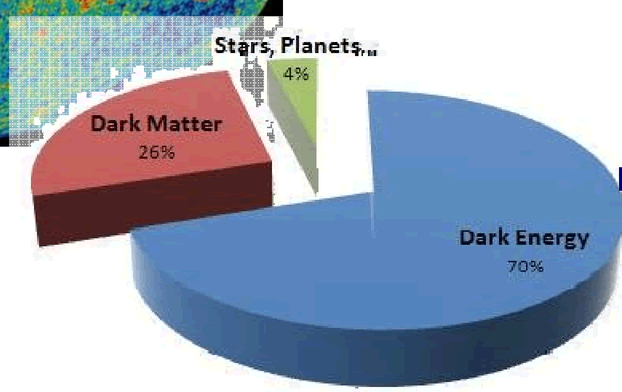
Matter In The Early Universe

- The history of luminous matter.
- Expansion + cooling; clumping and local reheating for the past ~ 13 billion years.
- Governed by gravity, dark matter, dark energy, …

[www.nasa.gov]



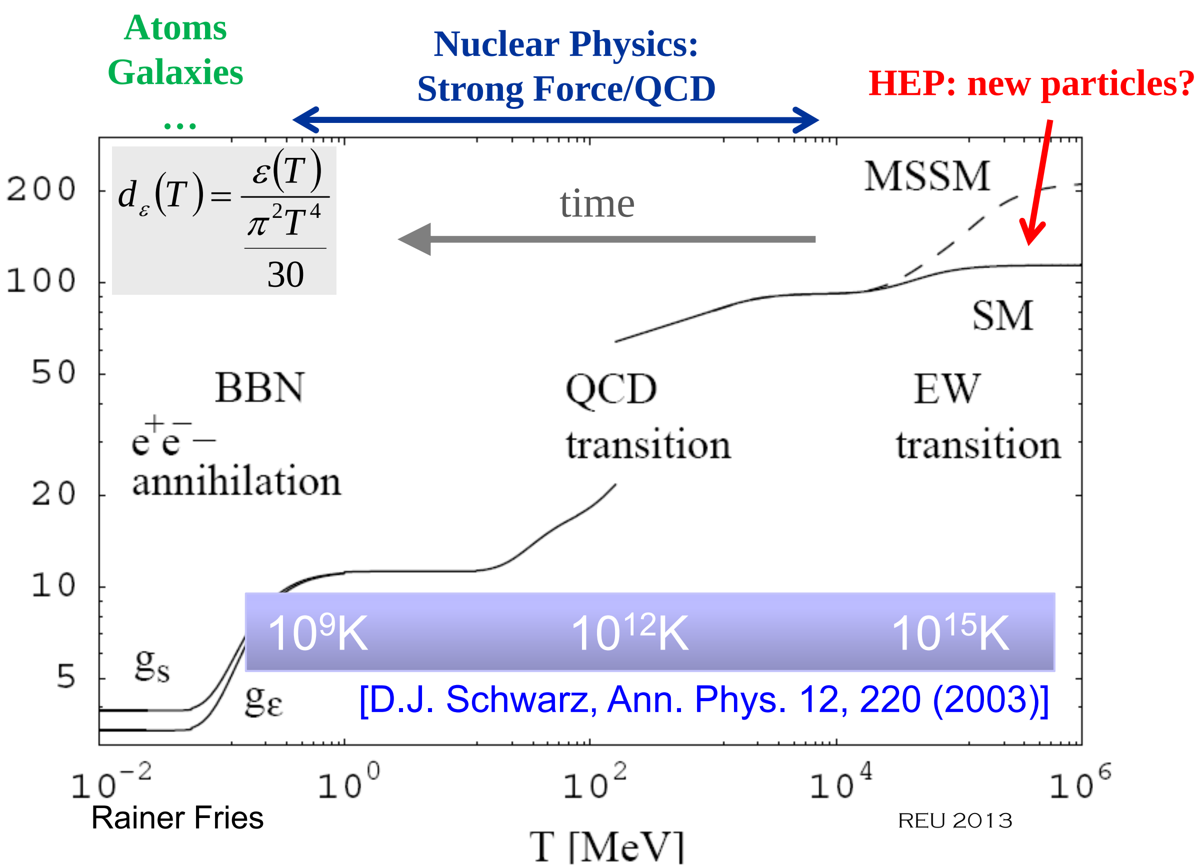


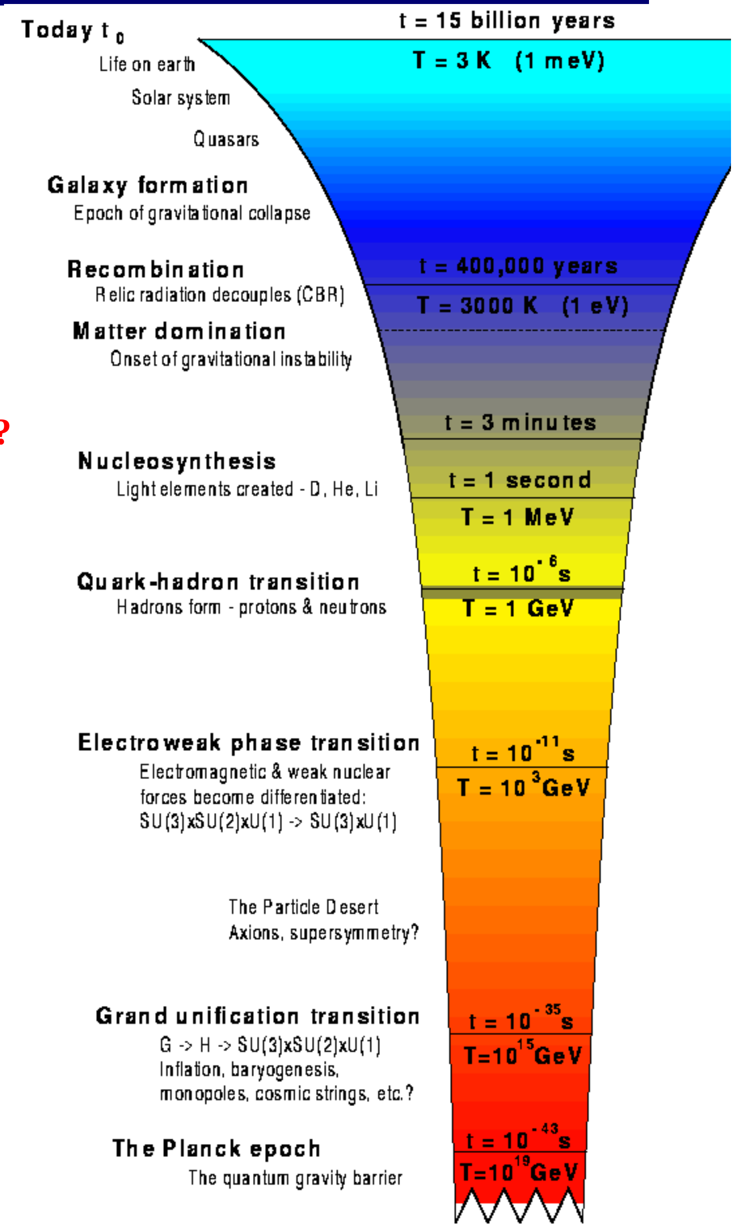


- The *tiny first second* of the universe: strong and electroweak forces are important!
- And maybe much more exotic stuff …

The First Second

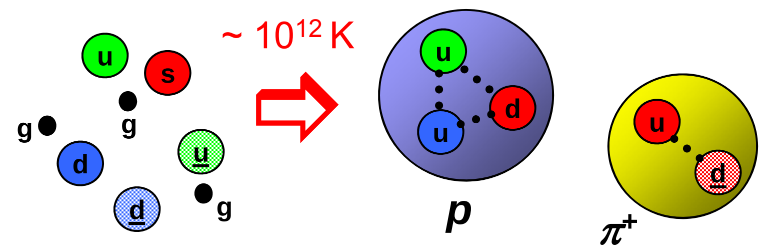
- Thermodynamic history: succession of phase transitions and freeze-outs.
- Rapid decrease in degrees of freedom.
- QCD transition @ ~ few μs



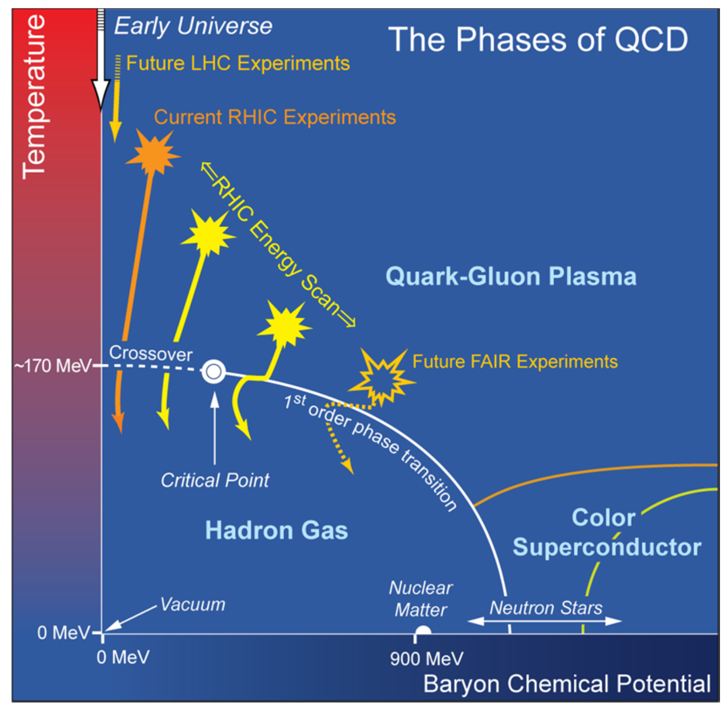


The QCD Transition in the Cosmos

- Bulk properties (thermodynamic, transport, …) of QCD played an important role in the early universe.
  - Order of the QCD phase transition, latent heat, speed of sound, …
- Enduring cosmic effects?
  - Initial conditions for nucleo-synthesis and beyond.
  - Mass generation for luminous matter.
  - Relics?



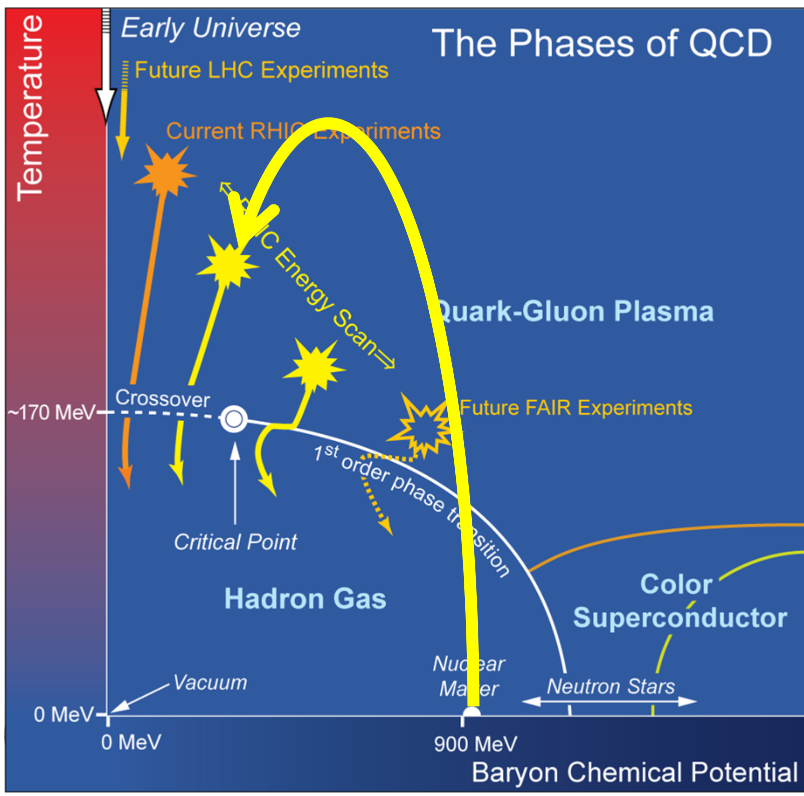
- How can QGP be studied experimentally today?

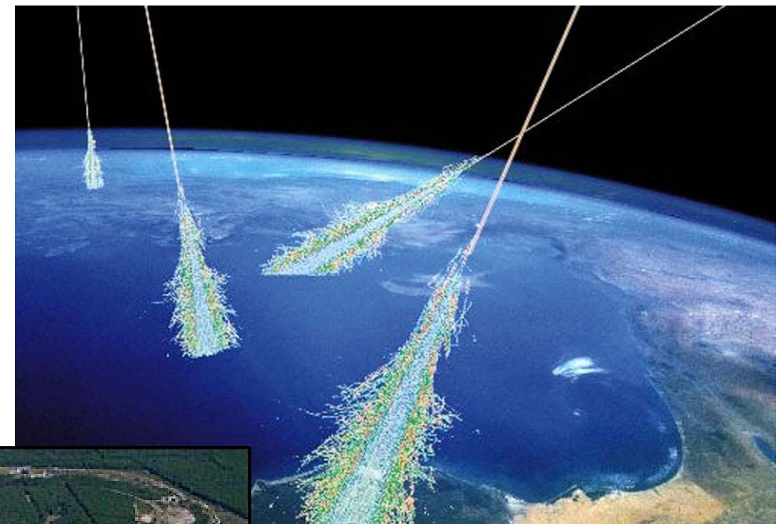


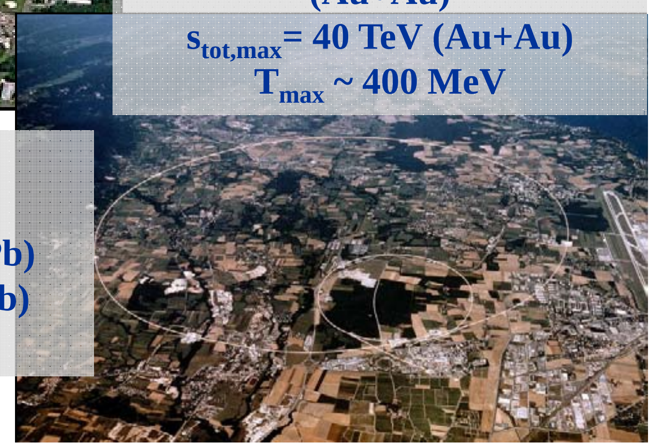
V. High Energy Heavy Ion Collisions

LET'S CREATE A 'LITTLE BANG'

- How can we study the QCD transition today?
  - Impacting cosmic rays (nuclear component!) – impractical
  - Nuclei in particle accelerators!
- Problem: extremely short life times ~ 10^{-23} s for the fireball ~ typical time scales of QCD.





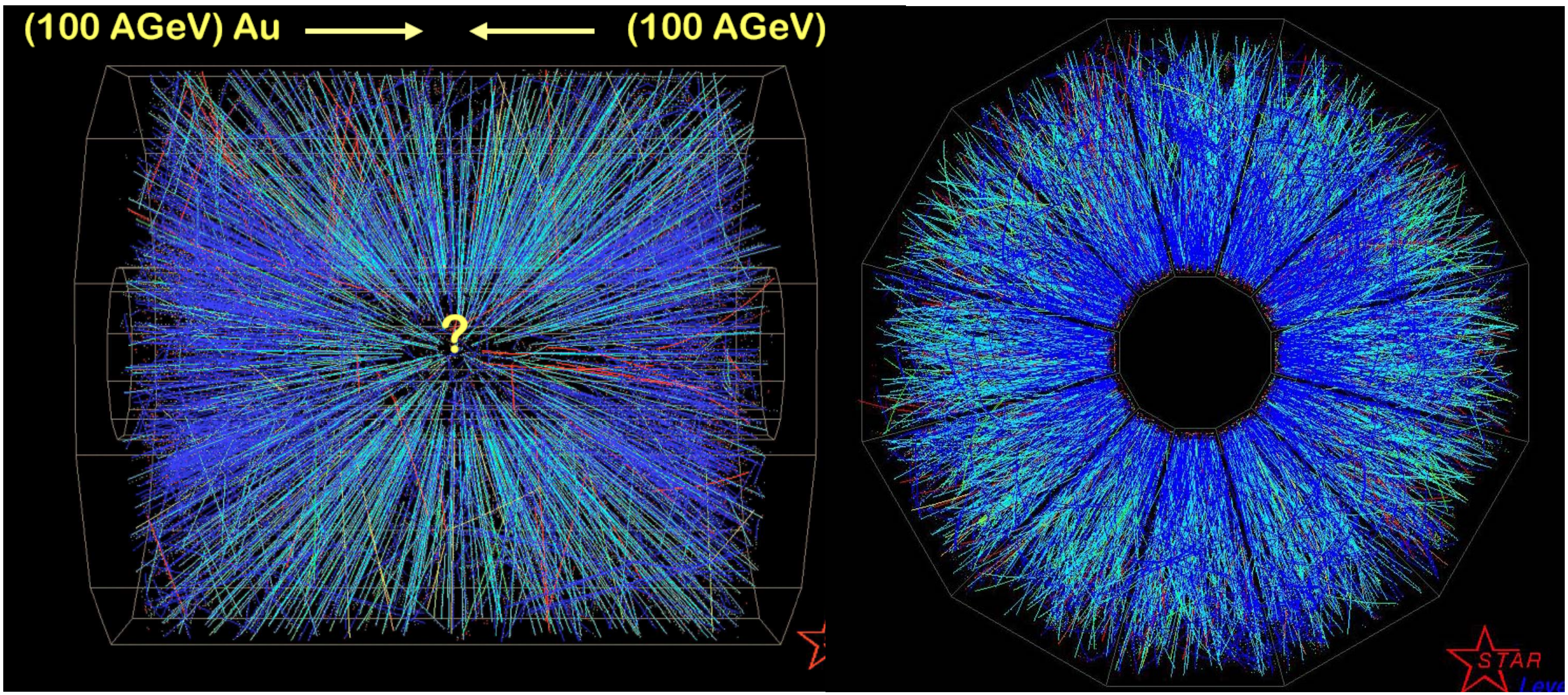


RHIC:
s_{NN} = 500 GeV (p+p)
s_{NN} = 200, 130, … 7.7 GeV (Au+Au)
$s_{tot,max}$ = 40 TeV (Au+Au)
T_{max} ~ 400 MeV

LHC:
s_{NN} = 14(7) TeV (p+p)
s_{NN} = 5.5(2.76) TeV (Pb+Pb)
s_{tot} = 1.1(0.55) PeV (Pb+Pb)
T_{max} ~ 800 MeV

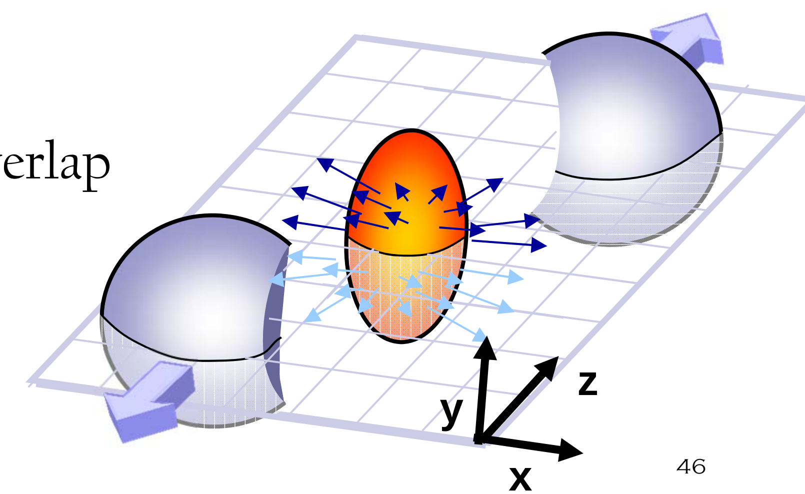
High Energy Heavy Ion Collisions (HICs)

- Thousands of particles created.
- Directed kinetic energy of beams → mass (particle) production + thermal motion + collective motion



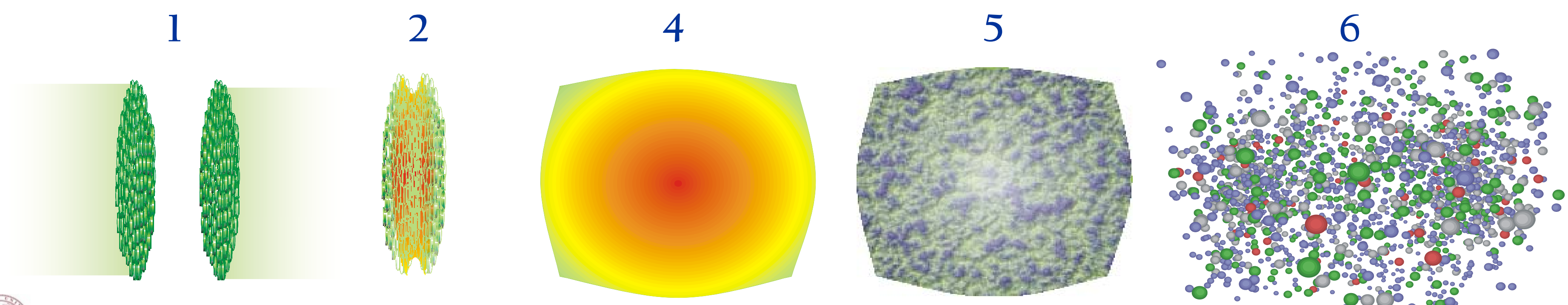
Basic Geometry

- Lorentz contraction of the nuclei $L \sim R/\gamma \to 0$.
- Approximate boost-invariance in beam direction a la Bjorken (later)
- Fireball: Longitudinal (~ boost invariant expansion) throughout time evolution
  - Not much altered through pressure gradients.
- Pressure in transverse expansion: collective transverse acceleration and expansion
- For arbitrary impact parameter *b*: elliptic overlap shape in transverse plane ("almond shape")
  - For non-spherial nuclei (e.g. U+U) many more geometrical degrees of freedom.



Time Evolution

1. Initial condition: nuclear wave functions
2. After nuclear overlap: no immediate thermalization of matter
   - Probably strong gluon fields/glasma.
3. Approx. thermal and chemical equilibrium reached after ~ 0.2 – 1.0 fm/c
   - Around midrapidity, checked through applicability of hydrodynamics.
4. QGP phase: initial temperatures up to ~400/600 MeV (RHIC/LHC)
   - Transverse expansion and cooling of the fireball
5. Hadronization around T_c and subsequent hot hadron gas phase
   - HRG may fall out of chemical equilibrium at "chemical freeze-out".
6. Decoupling of hadrons ("kinetic freeze-out") and free streaming of hadrons to detectors.

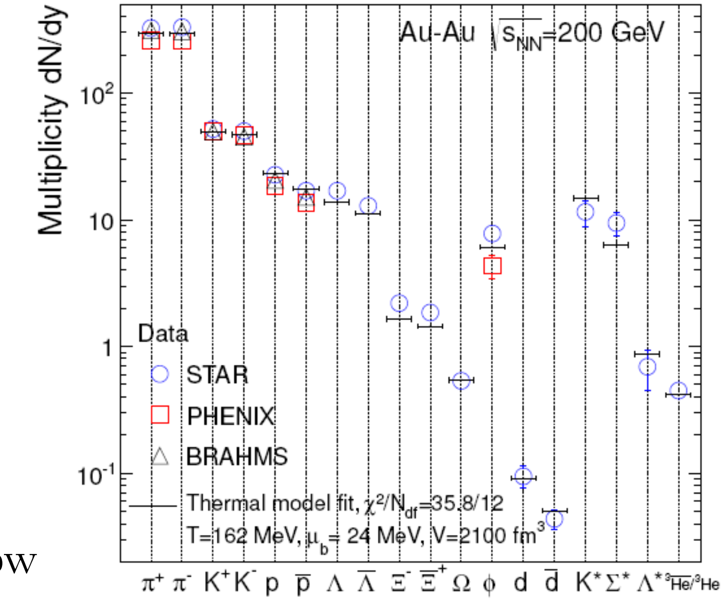


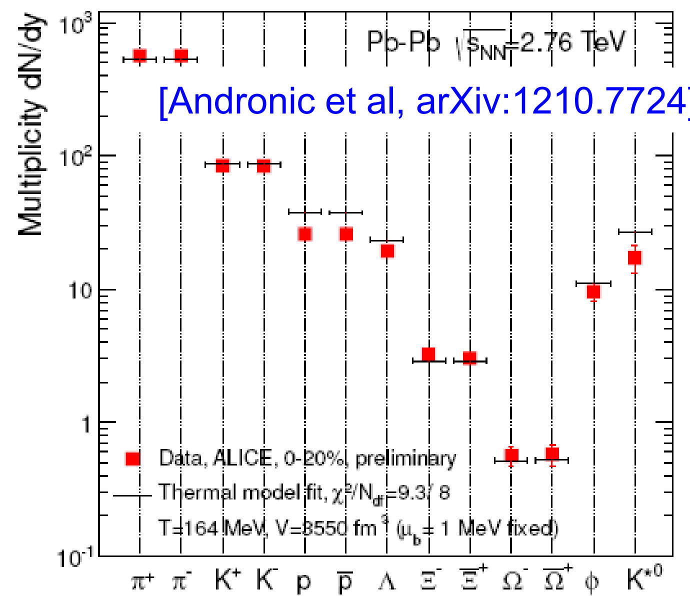
Chemical Equilibrium

- Hadrons are found in chemical equilibrium.
  - Compare to prediction from

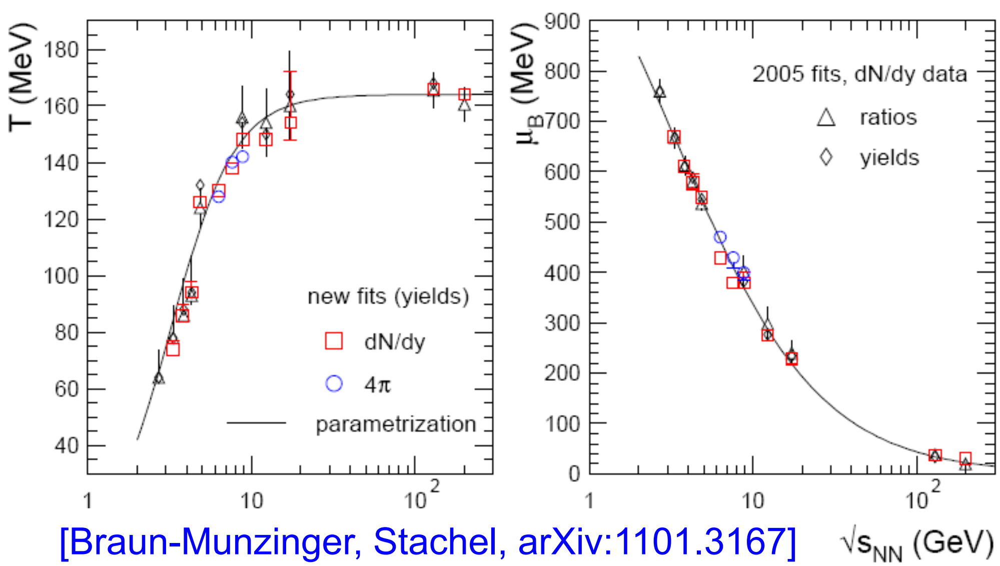
$$n_i(T,\mu) = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2 dp}{e^{(E_i-\mu_i B_i-\mu_s S_i)/T} \pm 1}$$

  - Chemical freeze-out temperature T_{chem} ~ 160-170 MeV
  - T_{chem} independent of $\sqrt{s}$ at large energies
  - Very small μ_B, compatible with dN_B/dy measurements.
- Enough time to chemically equilibrate!
  - But inelastic processes (e.g. K+p $\leftrightarrow$ π + Λ) shut off below T_{chem}; elastic processes needed for kinetic equilibrium!





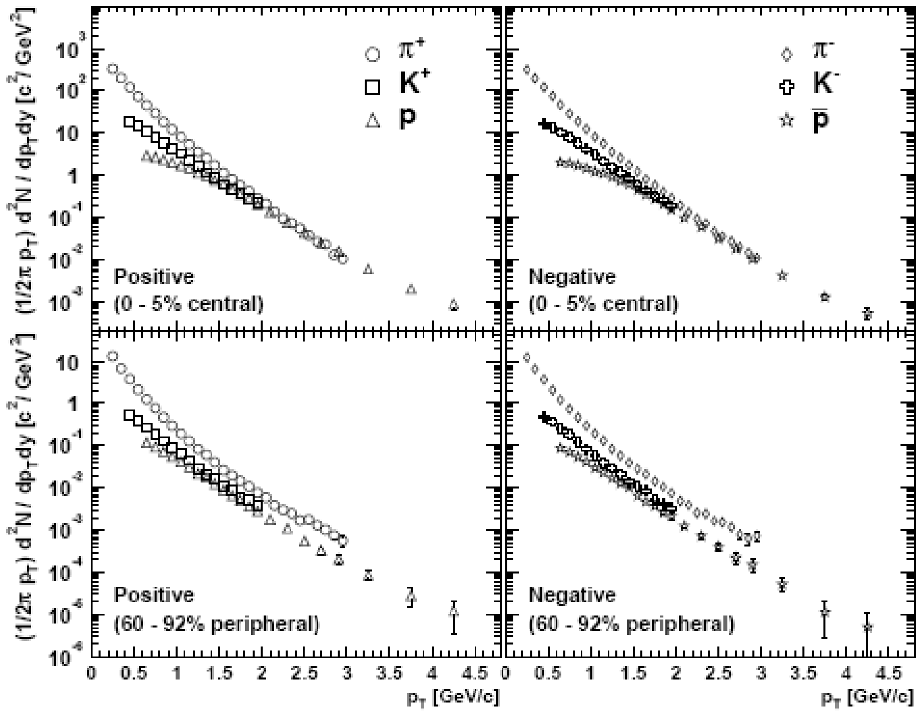
[Andronic et al, arXiv:1210.7724]

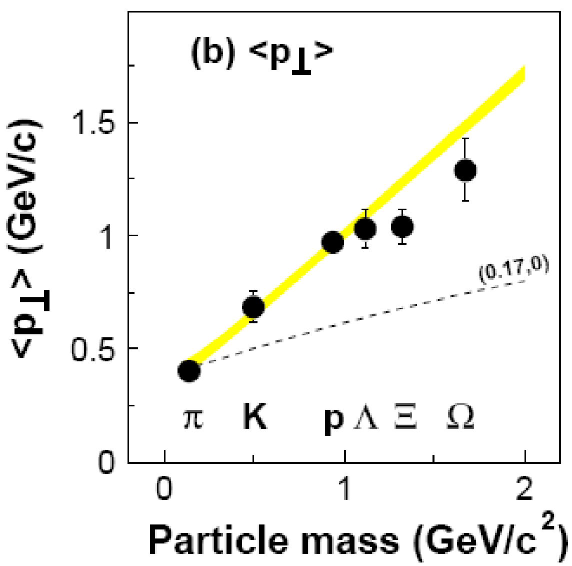


[Braun-Munzinger, Stachel, arXiv:1101.3167]

Thermal Equilibrium and Flow

- Another important test for equilibration: thermal transverse spectra of hadrons.
- However: thermal source is not at rest: collective transverse expansion.
  - Boltzmann with flow velocity u^μ:
    $e^{-E/T} \rightarrow e^{-p\cdot u/T}$
  - At low p_T: typical "flow shoulder" for heavier hadrons.
  - At high low p_T: blue shift of temperature

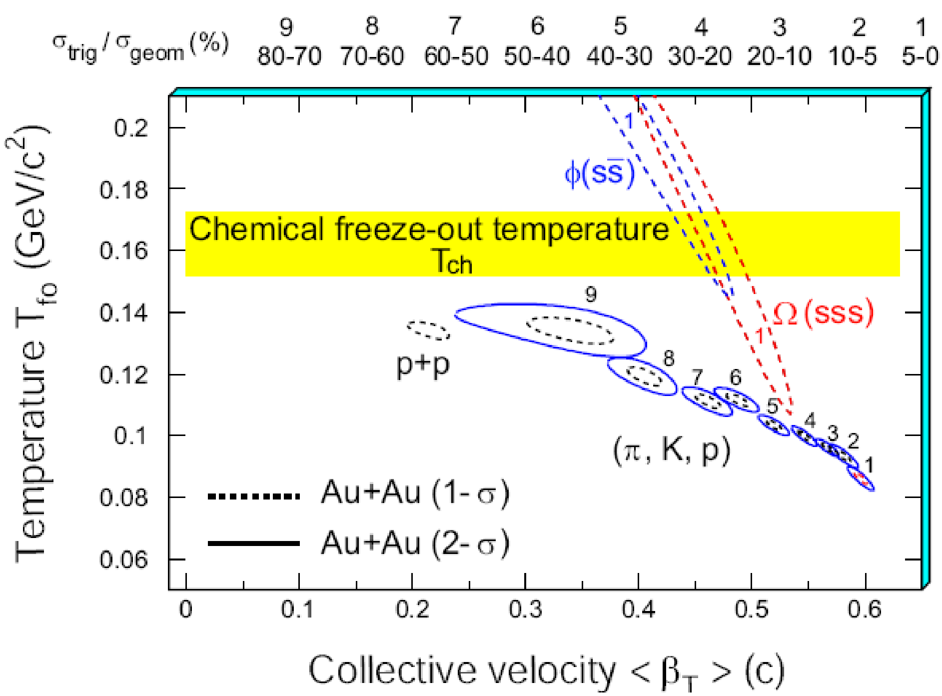




- Collective flow can also be observed in the average transverse momenta of different particles as a function of mass.

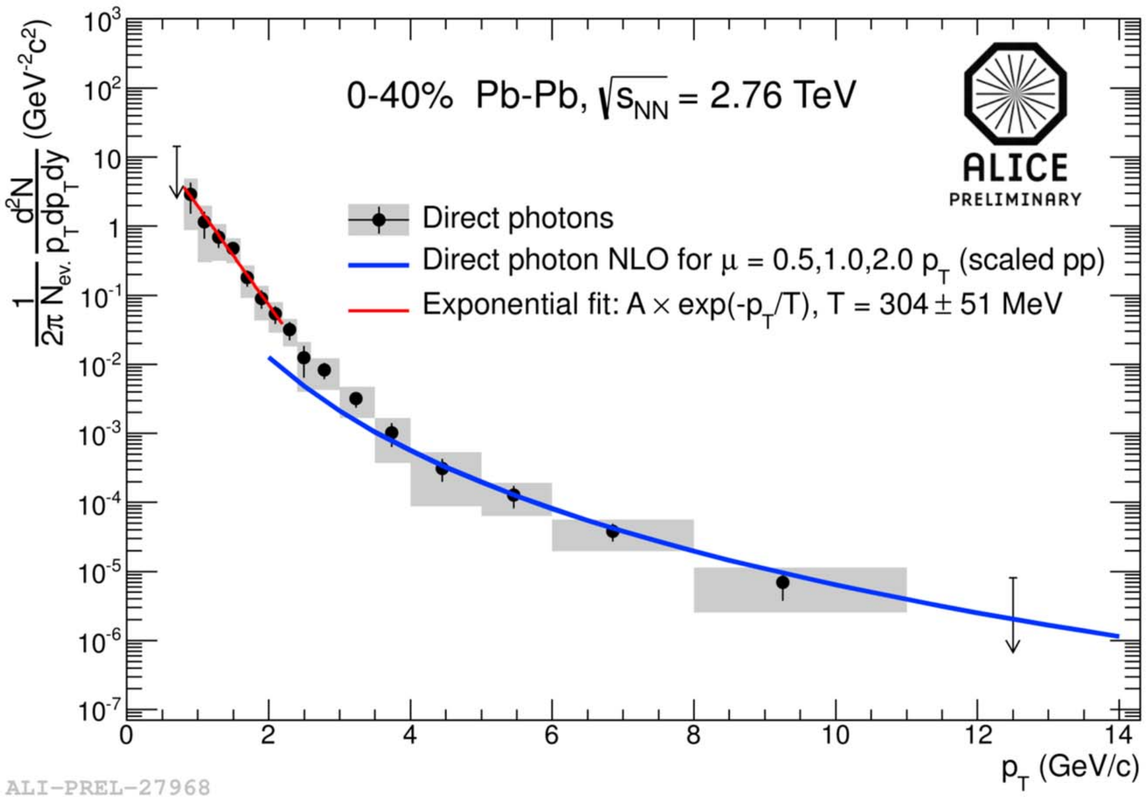
Flow and Freeze-out

- Bulk hadron data for p_T < 2 GeV can be fit well by "blast wave shape" = thermal distribution + flow.
  - Temperature in the fit = kinetic freeze-out temperature, typically ~100 MeV.
  - Typical average velocities 0.5-0.7 *c* for central RHIC and LHC.
- Kinetic equilibrium below T_{chem} is maintained by elastic scattering.
- Realistically: Not all particles decouple at the same temperature.
  - Furthermore: decoupling is not a sudden process but a gradual shut off of the interaction rate.
- This can be seen in higher freeze-out temperatures for multi-strange hadrons,e.g. ϕ, Ξ, Ω ("sequential freeze-out)
  - Small cross sections of these particles in a HRG.



Temperature Records

- We can measure the thermal radiation (blackbody) from the QGP phase.
- Photons do not feel the strong force: the fireball is almost transparent to them.
- Results from PHENIX and ALICE experiments.



Exponential fit for p_T < 2.2 GeV/c
inv. slope T = 304±51 MeV
for 0–40% Pb–Pb at √s 2.76 TeV
PHENIX: T = 221±19±19 MeV
for 0–20% Au–Au at √s 200 GeV

[Safarik, QM12]